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DOPPLER OBSERVABLE MODELING FOR THE APOLLO

REAL-TIME ORBIT DETERMINATION PROGRAM

Howard G. de Vezin

SUMMARY

This note presents a study of the current methods of computing the Doppler observable and, specifically, a range-difference technique for computing integrated Doppler count. The integrated Doppler count can be directly computed from the difference in lengths of the paths traversed by the transmitted signals at the beginning and end of the counting interval. When determining trajectories in the vicinity of the earth-moon system, this range-difference technique has certain advantages over an existing technique called the range-rate expansion method. There is no truncation error for the counting intervals presently planned for Apollo missions, and the formulation and resulting implementation are much shorter and less complex. Thus, this range-difference technique for computing the Doppler observable was chosen for use in the Apollo orbit determination program.

The derivation of the Doppler equation presented in this report employs Newtonian methods. In the computation of the position vectors of the vehicle and stations, the finite value of the velocity of light in the medium is taken into account. This approach is valid since, for two-way Doppler, the effects of general and special relativity on the observable in the earth-moon region are less than the known noise on the measurement; for three-way Doppler, these effects are significant only for widely separated stations and produce a small bias which may be solved for.

A numerical study was performed in which the total number of Doppler counts over the counting interval τ was computed using the range-rate expansion method. The truncation error was reduced by dividing the interval into smaller subintervals, computing the number of Doppler counts over each subinterval, and summing the results. This was taken as the standard value. From the same trajectory, at the same times, the Doppler count was computed using the range-difference method. The difference in the two values of Doppler count was then recorded.

Three cases were studied: earth orbit with a maximum elevation of 80°, early translunar, and lunar orbit. In the earth orbit case the maximum difference between the two methods was 6.0×10^{-5} cps or 1.4×10^{-5} fps in range-rate. In the translunar case the maximum difference was 2.0×10^{-3} cps or 0.5×10^{-3} fps. In the lunar orbit case both two-way and three-way Doppler were studied and the maximum difference was 3.0×10^{-5} cps or 0.7×10^{-5} fps. These values are orders of magnitude less than the instrument noise which for 6-second counting intervals is approximately 0.2 cps or 0.04 fps in range-rate.

These results show that, when the truncation error of the range-rate method has been removed, the two methods agree to well within measurement capabilities.

INTRODUCTION

Unified S-band Doppler count is the most fundamental and important ground-based observation used during Apollo missions for locating the spacecraft. The use of Doppler observable can be briefly described as follows. The Doppler count is recorded by the tracking stations. The real-time orbit determination program uses its current estimate of the vehicle's position and velocity to compute a corresponding value of Doppler count. The difference between the observed and computed values of Doppler count, called the residual, is computed and used by the orbit determination program to obtain an improved estimate of the vehicle's position and velocity.

The basic, continuous count Doppler observable is illustrated in figure 1. Two- and three-way Doppler geometry is shown. The transmitting

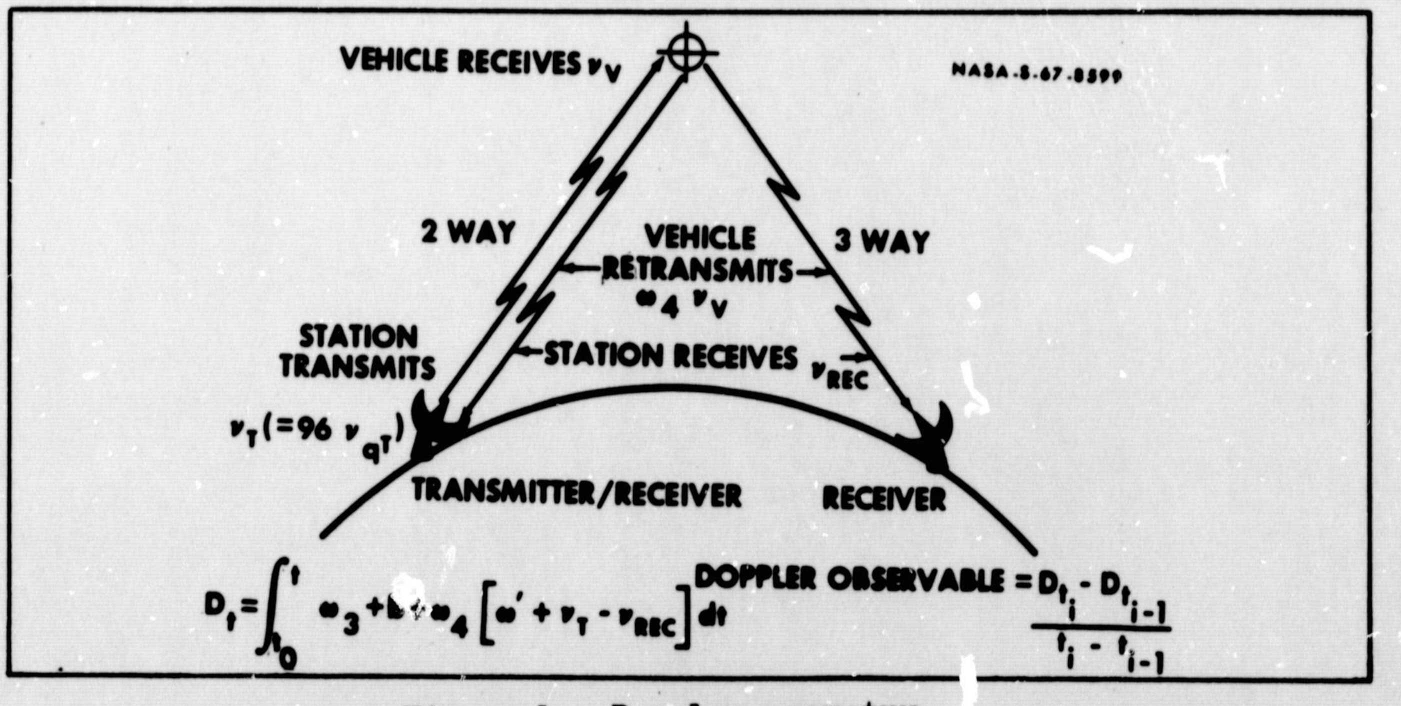


Figure 1.- Doppler geometry.

station transmits the frequency ν_T. The vehicle receives the signal, multiplies it by a constant, ω_4, and retransmits. The receiving station, which is the same as the transmitting station in the case of two-way Doppler and is a different station in the case of three-way Doppler, records the accumulated Doppler count.

The contents of this counter are recorded at precise intervals of time (usually either 6-second or 60-second counting intervals), and are transmitted to the real-time orbit determination program. After receiving this information, the difference between successive values of count is formed, and the result is normalized by dividing by the counting interval. This, then, is the expression that must be computed by the orbit determination program so that a residual can be formed, and an improved estimate of the vehicle trajectory obtained.

Range-Rate Expansion Method

To compute the expression, the integrated Doppler count over the counting interval, τ, must be computed. This can be accomplished by expanding the frequency shift in a power series of τ, and integrating term by term, evaluating the coefficients in the middle of the counting interval (ref. 1, 2, and 3). In the equations in references 1 and 2, only the first four terms of the expansion (i.e., the τ^0 and τ^2 terms) are carried (the τ^1 and τ^3 terms are zero).

It is pointed out in reference 4 that this power series expansion is adequate for presently planned τ intervals during the late translunar and lunar phases of Apollo type missions. However, it is also pointed out that in certain worst geometrical situations occurring in earth orbit and immediately after translunar injection, the errors resulting from the truncation in the τ series are significant.

This truncation error is illustrated in figure 2. In this case, an earth orbit pass with a maximum elevation of 87° is shown. The truncation error, expressed both in cycles per second and feet per second, is plotted versus elevation angle. The tracking station instrument errors are approximately 0.2 cps for a 6-second counting interval, and it can be seen that between the 60° elevation points, the truncation error is larger than the instrument error.

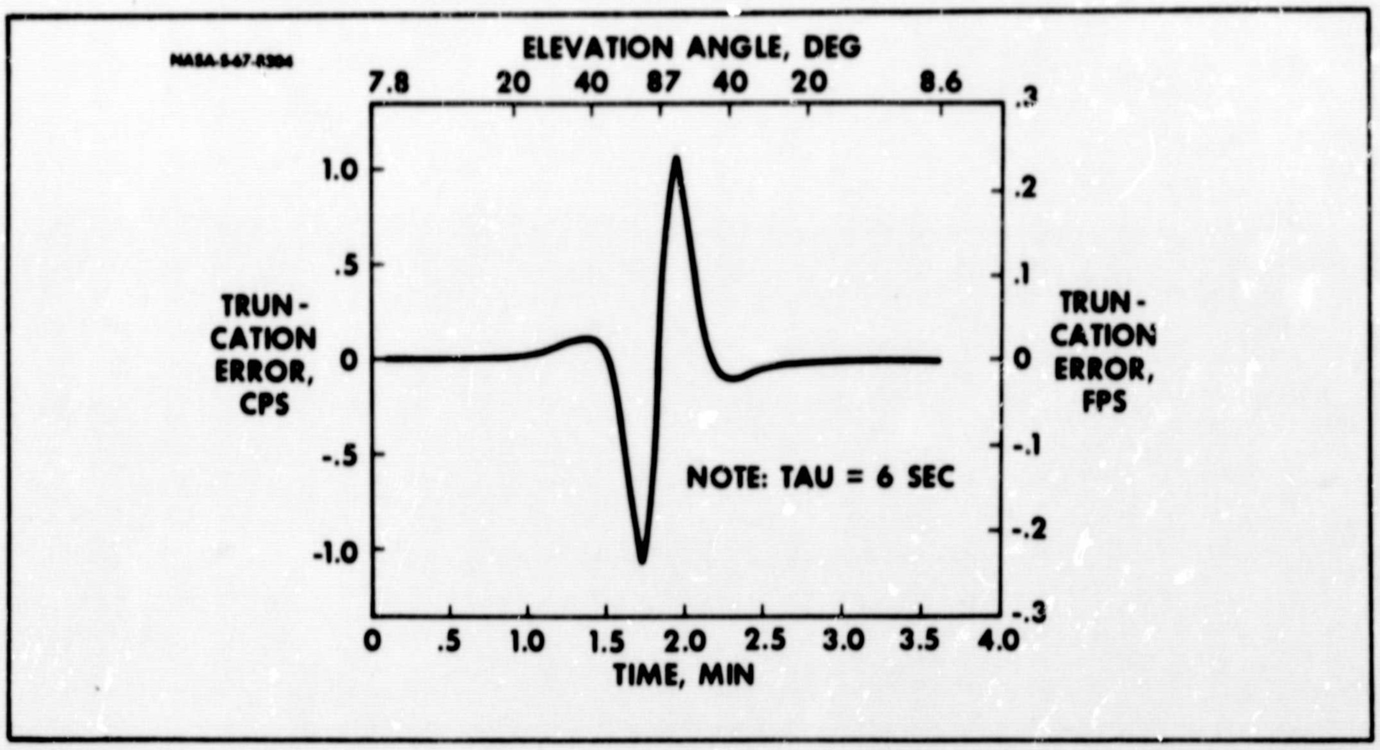


Figure 2.- Truncation error.

One method of avoiding the truncation error in these cases is to break the τ interval into several smaller intervals. For example, a 6-second τ interval could be broken into six, 1-second τ intervals. The expansion could then be applied to each of the smaller intervals, and the results summed to obtain the result for the entire τ interval. This, however, introduces mechanization problems into the orbit determination program and adds considerably to the computation time.

Range-Difference Method

Another method of computing the Doppler observable is discussed in references 5 through 9. This range-difference technique computes the integrated Doppler count directly from the difference in lengths of the paths traversed by the transmitted signals at the beginning and end of the counting interval.

There are two main advantages of the range-difference method over the previously discussed technique. First, since no expansion in powers of τ is made, no truncation errors proportional to the length of τ exist. Secondly, the formulation and resulting implementation are much shorter and less complex than is the case with the approach which uses an expansion in τ.

The main disadvantage of this approach is that, in lunar orbit, several significant digits are lost when a range-difference computation is made. This is due to the large range values, of the order 4×10^{10} cm, which exist in lunar orbit. However, when the computations are performed in double precision this truncation effect in the range-difference equation is not significant.

A derivation of the equations for computing integrated Doppler count using the range-difference technique is discussed in detail in reference 9. A more direct approach is presented in this report. In both developments of the Doppler equation, Newtonian methods are employed and the finite value of the velocity of light in the medium is taken into account when computing the position vectors of the vehicle and stations.

This approach is valid since for two-way Doppler, the effects of general and special relativity on the observable in the earth-moon region, and for vehicle velocities involved, are less than the known noise on the measurement, and for three-way Doppler these effects are significant only for widely separated stations and produce a small bias which may be solved for.

In view of the advantages of the range-difference technique over the method which uses the expansion in τ and since the Apollo orbit determination program is in double precision, the range-difference technique for computing the Doppler observable was chosen for use in the Apollo orbit determination programs.

The main text of this note presents the derivation of the Doppler equation. The light-time delay iterative technique, which is used to compute the position vectors of the vehicle and the transmitting station, is then discussed. Finally, the partial derivatives of the Doppler observable with respect to the state and biases are presented.

SYMBOLS

b	unknown Doppler bias which may be solved for
c	speed of light
D	integrated Doppler count observable
D_{t_R}	accumulated Doppler count from time t_o to time t_R
$D_{t_R-\tau}$	accumulated Doppler count from time t_o to time $t_R-\tau$

$S_R = \begin{bmatrix} R_R \\ \dot{R}_R \end{bmatrix}$	state vector of the receiving station in earth-centered inertial coordinates
$S_T = \begin{bmatrix} R_T \\ \dot{R}_T \end{bmatrix}$	state vector of the transmitting station in earth-centered inertial coordinates
$S_v = \begin{bmatrix} R_v \\ \dot{R}_v \end{bmatrix}$	vehicle state vector in earth-centered inertial coordinates obtained from the vehicle ephemeris
t_o	the last time at which the Doppler counter at the receiving station was initialized
t_R	time at which the signal was received at the receiving station
t_T	time at which the signal left the transmitting station
t_v	time at which the signal arrived at, and was transmitted by the vehicle
Δt_t	propagation time for signal which reaches receiving station at time t
ϕ	instantaneous phase
ν_{REC}	instantaneous received frequency
ν_T	instantaneous transmitted frequency
ρ_1	range value from the transmitter to the vehicle
ρ_2	range value from the receiver to the vehicle
τ	Doppler count sampling interval
ω'', ω'	the Doppler bias which will result, in the case of three-way Doppler, when the standard frequencies of the two stations are set to different values
ω_3	a large positive constant frequency, added to the incoming frequency at the receiving station, which is designed to keep the Doppler count positive over the sampling interval
ω_4	a frequency multiplying factor by which the transponder in the spacecraft alters the frequency received by the spacecraft.

DERIVATION OF DOPPLER EQUATION

A derivation of the Doppler equation is presented in reference 9. However, the development presented below is more straight forward and more closely describes the functions of the Doppler count measuring equipment at the receiving station.

The analytical expression which describes the accumulated Doppler count, D_t, measured at the receiving station is

$$D_t = \int_{t_o}^{t} \omega_3 + b + \omega_4\left(\omega'' + \nu_T - \nu_{REC}\right) dt \qquad (1)$$

The difference between successive values of count, $D_{t_i} - D_{t_{i-1}}$, is formed in the orbit determination program. This is then normalized by dividing by the counting interval $\tau = t_i - t_{i-1}$. Thus

$$D = \frac{D_{t_R} - D_{t_R-\tau}}{\tau} = \omega_3 + b + \omega' + \frac{1}{\tau}\int_{t_R-\tau}^{t_R} \omega_4\left(\nu_T - \nu_{REC}\right)_{REC} dt, \qquad (2)$$

where the values ν_T and ν_{REC} are instantaneous values of frequency and both are expressed at the same instant of time, measured at the receiving station, and the counting interval, τ, begins at time $t_R-\tau$ and ends at time t_R.

The frequency can be expressed in terms of phase by the expression

$$\nu = \frac{d\phi}{dt} \quad . \qquad (3)$$

Equations (2) and (3) can be combined to form the following expression

$$D = \omega_3 + b + \omega' + \frac{\omega_4}{\tau}\int_{t_R-\tau}^{t_R} \left(\frac{d\phi_T}{dt} - \frac{d\phi_{REC}}{dt}\right)_{REC} dt. \qquad (4)$$

Equation (4) can be written as

$$D = \omega_3 + b + \omega' + \frac{\omega_4}{\tau} \int_{t_R - \tau}^{t_R} d\left(\phi_T - \phi_{REC}\right)_{REC} . \tag{5}$$

An exact evaluation of the integral in equation (5) can be obtained. This results in equation (6) which expresses the integrated Doppler count in terms of the instantaneous phase difference as measured at the receiving station at the beginning and ending times of the counting interval.

$$D = \omega_3 + b + \omega' + \frac{\omega_4}{\tau} \left[\left(\phi_T - \phi_{REC}\right)_{t_R} - \left(\phi_T - \phi_{REC}\right)_{t_R - \tau}\right] . \tag{6}$$

The instantaneous phase difference must be evaluated in terms of tracker, vehicle geometry.

For a particular receiver time, t, it has taken a finite amount of time for the signal to have traveled from the transmitter to the receiver. It is assumed that phase propagates, unaltered, through the medium.

At time, t, the receiver phase will, therefore, differ from the transmitter phase by the amount that the transmitter phase has changed during the propagation time.

In equation 7, it can be seen that the amount that the transmitter phase has changed is given by the transmitter frequency, ν_T, multiplied by the propagation time Δt_t,

$$\left(\phi_T - \phi_{REC}\right)_t = \nu_T \Delta t_t \quad . \tag{7}$$

The propagation time is equal to the total distance, ρ_t, traveled by the signal which reaches the receiver at time t, divided by the velocity of light

$$\Delta t_t = \frac{\rho_t}{c} \quad .$$

The total distance traveled by the signal is the sum of the uplink distance, ρ_1, and the downlink distance, ρ_2,

$$\rho_t = \left(\rho_1 + \rho_2\right)_t \quad .$$

Combining these expressions, one obtains the instantaneous phase difference in terms of the transmitted frequency and the up and downlink distances that were traveled by the signal which arrived at the receiver at time t,

$$\left(\phi_T - \phi_{REC}\right)_t = \nu_T \frac{\left(\rho_1 + \rho_2\right)_t}{c} \quad . \tag{8}$$

Substituting equation (8) into equation (6), the final result can be obtained which expresses the Doppler observable in terms of the paths traveled by the signal at the beginning and ending times of the counting interval,

$$D = \omega_3 + b + \omega' + \frac{\nu_T \omega_4}{\tau c}\left[\left(\rho_1 + \rho_2\right)_{t_R} - \left(\rho_1 + \rho_2\right)_{t_R - \tau}\right] \quad . \tag{9}$$

The method used to compute the distance traveled by the signal is presented in the next section.

LIGHT-TIME DELAY

Figure 3 shows a signal which leaves the transmitter at time t_T, travels a distance ρ_1, and reaches the vehicle at time t_v. The vehicle

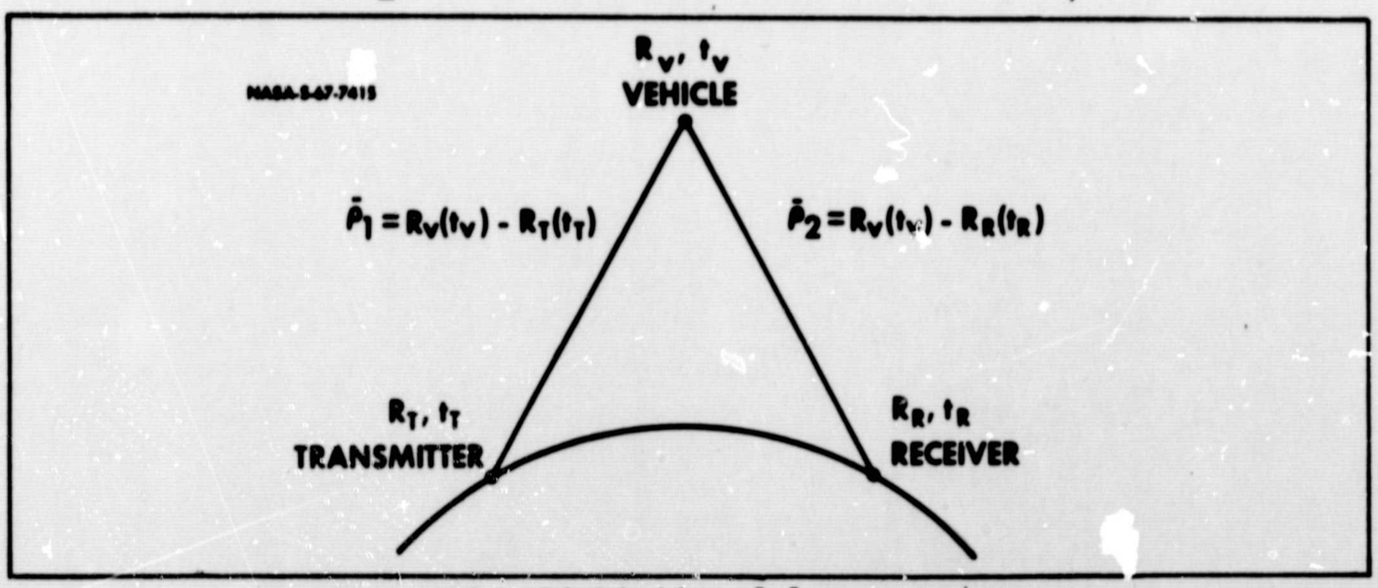


Figure 3.- Light time-delay geometry.

at this time has a position R_v. The signal then travels a distance ρ_2 and is received by the receiving station at t_R.

Because the velocity of light is finite, t_R, t_v, and t_T are related by the expressions

$$t_v = t_R - \frac{\rho_2}{c} \tag{10a}$$

and

$$t_T = t_R - \frac{\rho_1 + \rho_2}{c} \quad . \tag{10b}$$

The time at which the signal is received, t_R, is known. The ray path lengths ρ_1 and ρ_2 can be determined from the position vectors R_R, R_v, and R_T as functions of t_R, t_v, and t_T, respectively, by iterating on the expressions

$$\rho_2 = |R_v(t_v) - R_R(t_R)| \tag{11a}$$

$$\rho_1 = |R_v(t_v) - R_T(t_T)| \tag{11b}$$

For the first iteration the position vectors R_v, R_R, and R_T are computed for time t_R. These values are used to obtain ρ_1 and ρ_2, and t_v and t_T are then computed from equation 10.

These values of t_v and t_T are used to obtain better estimates of R_v and R_T, and, hence, better estimates of ρ_1 and ρ_2. This iterative process continues until both t_v and t_T are within specified tolerances. The vector R_v is obtained from the vehicle ephemeris using Lagrange interpolation. A detailed discussion of this iterative procedure is presented in reference 10.

This method is used to compute the total distance traveled by the signals which reach the receiving station at the beginning and ending times of the counting interval. These values are then used in equation (9) to obtain the integrated Doppler count observable.

DOPPLER PARTIALS

To compute the Doppler partials the assumption is made that

$$\frac{\left(\rho_1 + \rho_2\right)_{t_R} - \left(\rho_1 + \rho_2\right)_{t_R-\tau}}{\tau} \simeq \frac{d}{dt}\left(\rho_1 + \rho_2\right)_{t_R-\frac{\tau}{2}} \quad . \qquad (12)$$

Using the light-time delay technique, the vehicle ephemeris, and the position vector of the receiving station at $t_R - \frac{\tau}{2}$ as input, the following quantities are computed:

$$\rho_1,\ \rho_2,\ R_v\left(t_R - \frac{\rho_2}{c} - \frac{\tau}{2}\right),\ \dot{R}_v\left(t_R - \frac{\rho_2}{c} - \frac{\tau}{2}\right),$$

$$R_T\left(t_R - \frac{\rho_1 + \rho_2}{c} - \frac{\tau}{2}\right),\ \dot{R}_T\left(t_R - \frac{\rho_1 + \rho_2}{c} - \frac{\tau}{2}\right),$$

$$\dot{R}_R\left(t_R - \frac{\tau}{2}\right) .$$

To simplify the terminology, the time subscripts will be omitted. Using the approximation made in equation (12), equation (9) becomes

$$D = \omega_3 + b + \omega' + \frac{\omega_4 \nu_T}{c}\left[\frac{d}{dt}\left(\rho_1 + \rho_2\right)\right]$$

or

$$D = \omega_3 + b + \omega' + \frac{\omega_4 \nu_T}{c}\left(\frac{d\rho_1}{dt} + \frac{d\rho_2}{dt}\right). \qquad (13)$$

Now $\rho_1 = |R_v - R_T|$, and for any vector $A = a\hat{A}$, we have the relationship $A \cdot \frac{dA}{dt} = a\,\frac{da}{dt}$. Therefore,

$$\frac{d\rho_1}{dt} = \frac{(R_v - R_T) \cdot (\dot{R}_v - \dot{R}_T)}{\rho_1} = \frac{R_v \cdot (\dot{R}_v - \dot{R}_T) - R_T \cdot (\dot{R}_v - \dot{R}_T)}{\rho_1} \qquad (14a)$$

and, similarly,

$$\frac{d\rho_2}{dt} = \frac{(R_v - R_R) \cdot (\dot{R}_v - \dot{R}_R)}{\rho_2} = \frac{R_v \cdot (\dot{R}_v - \dot{R}_R) - R_R \cdot (\dot{R}_v - \dot{R}_R)}{\rho_2} . \quad (14b)$$

From equation (13) we have

$$\frac{\partial D}{\partial R_v} = \frac{\omega_4 \nu_T}{c}\left[\frac{\partial}{\partial R_v}\left(\frac{d\rho_1}{dt} + \frac{d\rho_2}{dt}\right)\right]. \quad (15)$$

From equation (14), we have

$$\frac{\partial}{\partial R_v}\left(\frac{d\rho_1}{dt}\right) = \frac{\dot{R}_v - \dot{R}_T}{\rho_1} + \frac{-1(R_v - R_T) \cdot (\dot{R}_v - \dot{R}_T)}{\rho_1^2} \frac{\partial \rho_1}{\partial R_v} .$$

Now

$$\frac{\partial \rho_1}{\partial R_v} = \frac{\partial}{\partial R_v} \left|R_v - R_T\right| = \frac{\partial}{\partial R_v}\left[(R_v - R_T) \cdot (R_v - R_T)\right]^{\frac{1}{2}}$$

or

$$\frac{\partial \rho_1}{\partial R_v} = \frac{1}{2}\left[(R_v - R_T) \cdot (R_v - R_T)\right]^{-\frac{1}{2}} \frac{\partial}{\partial R_v}(R_v \cdot R_v - 2R_v \cdot R_T + R_T \cdot R_T)$$

and, therefore,

$$\frac{\partial \rho_1}{\partial R_v} = \frac{R_v - R_T}{\rho_1} .$$

Therefore, we have

$$\frac{\partial}{\partial R_v}\left(\frac{d\rho_1}{dt}\right) = \frac{\dot{R}_v - \dot{R}_T}{\rho_1} - \frac{(R_v - R_T) \cdot (\dot{R}_v - \dot{R}_T)}{\rho_1^2}\left(\frac{R_v - R_T}{\rho_1}\right) ,$$

and using equation (14), this becomes

$$\frac{\partial}{\partial R_v}\left(\frac{d\rho_1}{dt}\right) = \frac{\dot{R}_v - \dot{R}_T}{\rho_1} - \frac{1}{\rho_1^2}\frac{d\rho_1}{dt}\left(R_v - R_T\right)$$

Similarly, it can be shown that

$$\frac{\partial}{\partial R_v}\left(\frac{d\rho_2}{dt}\right) = \frac{\dot{R}_v - \dot{R}_R}{\rho_2} - \frac{1}{\rho_2^2}\frac{d\rho_2}{dt}\left(R_v - R_R\right)$$

Substituting these equations into equation (15) we have

$$\frac{\partial D}{\partial R_v} = \frac{\omega_4 \nu_T}{c}\left[\frac{\dot{R}_v - \dot{R}_T}{\rho_1} - \frac{1}{\rho_1^2}\frac{d\rho_1}{dt}\left(R_v - R_T\right) + \frac{\dot{R}_v - \dot{R}_R}{\rho_2} - \frac{1}{\rho_2^2}\frac{d\rho_2}{dt}\left(R_v - R_R\right)\right].$$

From equation (14) we see that

$$\frac{\partial}{\partial \dot{R}_v}\left(\frac{d\rho_1}{dt}\right) = \frac{R_v - R_T}{\rho_1}$$

and

$$\frac{\partial}{\partial \dot{R}_v}\left(\frac{d\rho_2}{dt}\right) = \frac{R_v - R_R}{\rho_2}$$

Therefore, we have the result that

$$\frac{\partial D}{\partial \dot{R}_v} = \frac{\omega_4 \nu_T}{c}\left[\frac{R_v - R_T}{\rho_1} + \frac{R_v - R_R}{\rho_2}\right].$$

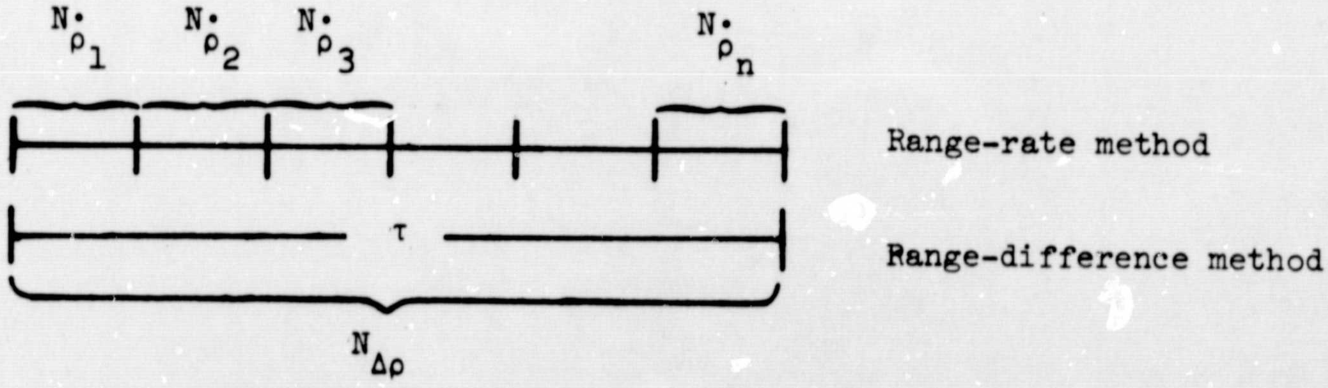
It can be shown that these results and the approximations involved are the same as those given in reference 3.

From equation (9), it is seen that the partial of the observation with respect to the bias is given by

$$\frac{\partial D}{\partial b} = 1 \quad .$$

NUMERICAL RESULTS

The procedure used to numerically verify the range-difference method is shown below.



The total number of Doppler counts over the counting interval τ, $N_{\dot{\rho}} = N_{\dot{\rho}_1} + N_{\dot{\rho}_2} + N_{\dot{\rho}_3} + \ldots + N_{\dot{\rho}_n}$, was computed using the range-rate expansion method. The truncation error was reduced by dividing the interval into smaller subintervals, computing the number of Doppler counts over each subinterval ($N_{\dot{\rho}_1}$, $N_{\dot{\rho}_2}$, ..., $N_{\dot{\rho}_n}$), and summing the results.

This was taken as the standard value. From the same trajectory at the same times, the Doppler count was computed using the range-difference method ($N_{\Delta\rho}$). The difference in the two values of Doppler, $\Delta N = N_{\dot{\rho}} - N_{\Delta\rho}$ was then recorded.

Three trajectories were studied: earth orbit with a maximum elevation of 80°, early translunar, and lunar orbit. In the lunar orbit case both two-way and three-way Doppler were studied.

The maximum values of ΔN for these cases are presented in the following table.

Case	Difference, cps (fps)	τ, sec	n
Earth orbit	6.0×10^{-5} (1.4×10^{-5})	6.0	60
Translunar orbit	2.0×10^{-3} (0.5×10^{-3})	6.0	6
Lunar orbit (2-way and 3-way)	3.0×10^{-5} (0.7×10^{-5})	60.0	60

In all cases the values of ΔN are orders of magnitude less than the instrument noise, which for 6-second counting intervals, is approximately 0.2 cps or .04 fps in range-rate.

The truncation error in the range-rate Doppler equation was also studied. This was done by comparing the Doppler count obtained by summing six consecutive 1-second count values, with the count obtained by using a counting interval of 6 seconds. In both cases, Doppler was computed using the range-rate equation. An earth orbit trajectory, with a maximum elevation of 87°, was used. The truncation error, illustrated in figure 2, is expressed both in cycles per second and feet per second and is plotted versus elevation angle. From figure 2, it can be seen that, the truncation error, between the 60° elevation points, is larger than the instrument error (0.2 cps).

CONCLUSIONS

Two conclusions can be drawn from the numerical results. First, a significant truncation error exists in the range-rate Doppler equation for certain earth orbit geometry situations when presently planned Apollo

mission counting intervals are used. Second, when the truncation error of the range-rate method has been removed, the range difference Doppler agrees with the adjusted range-rate value to well within measurement capabilities.

The advantages of the range-difference method are

1. No truncation error exists.

2. Formulation and resulting implementation are less complex than the range rate expansion method.

3. This method gives more physical insight into the Doppler observable and the various phenomena which affect it, e.g., atmospheric refraction, station location errors, and time tagging errors.

The limitations of the range-difference method are

1. Computations must be performed in double precision for lunar distances.

2. Double precision would be marginal for deep space missions.

In view of the advantages of the range-difference method, it was chosen for use in the Apollo real-time orbit determination program.

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